**CS3081 – Computational Mathematics**

**Recommended Questions Solutions**



Consider a function that is differentiable times in an interval containing the point .

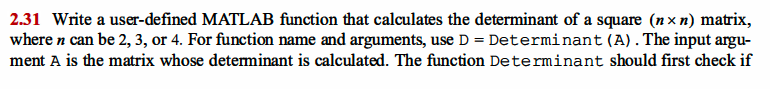
The Taylor theorem states that for each in the interval, there exists a value between and such that:

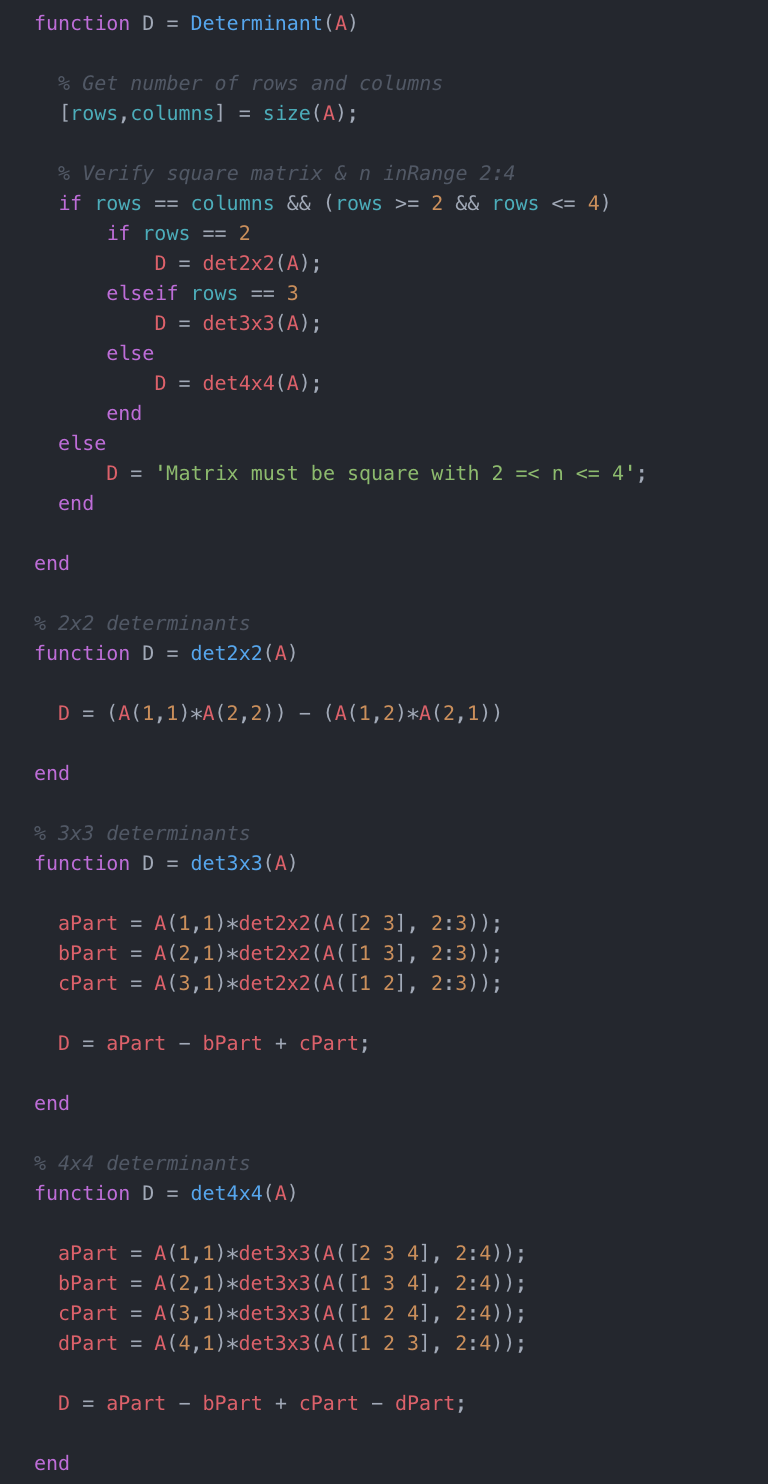
Where called the remainder and is given by:

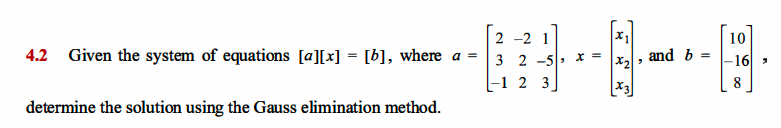
The Taylor series expansion of the function about (i.e ) is therefore as follows:

Substitute into this function:

Therefore, the Taylor’s series expansion of the function is:







First, substitute the given matrices in form:

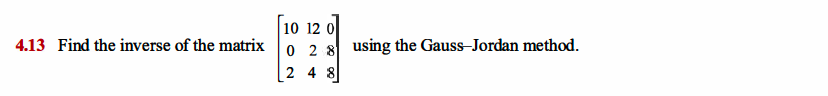
We then want to eliminate the all of the bottom row ( elements besides the 3 so that we can get a value for x3. Therefore perform the following row operations on matrices and :

* - 3()
* +
* -

From this we can now solve for :

We can then sub this into row 2 and solve for

And then solve for using row 1:

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First write out the matrix and the augmented matrix:

The aim of the game here is to reduce the LHS matrix to the RHS matrix by performing row operations:



We know from 4.13 that the inverse of a is:

Consider the formula for the condition number of a matrix:

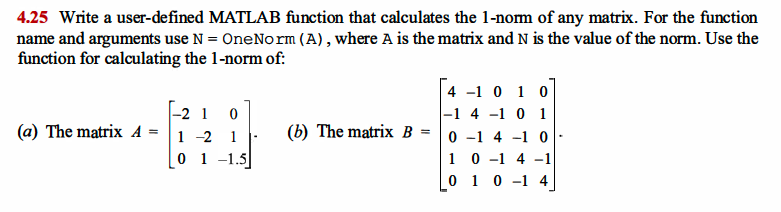
Where:

* denotes the norm of matrix a
* denotes the norm of inverse of matrix a

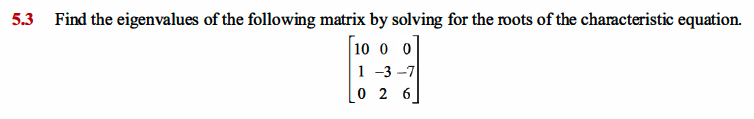
Write the formula to find the 1-norm of a matrix:

Consider the matrix a as stated above and 3 for n:

Therefore, the 1-norm of matrix a is 18.







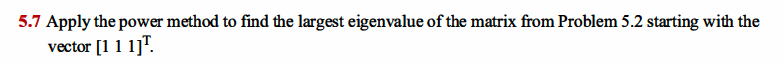
The expression for the characteristic equation to find the Eigen values is:

As the matrix a is of order 3x3, the order of the identity matrix is also 3x3:

Calculate the matrix :

Using this calculate the characteristic equation:

Therefore, the Eigen values of the matrix are 10, 4 and -1



For , the Eigen vector that is:

Calculate the next Eigen vector for

We must then extract the highest element from the vector and normalise the vector:

Therefore, the normalized unit vector of is

We then repeat these steps, for the next Eigen vector for

We must then extract the highest element from the vector and normalise the vector:

Therefore, the normalized unit vector of is

We then repeat these steps, for the next Eigen vector for

We must then extract the highest element from the vector and normalise the vector:

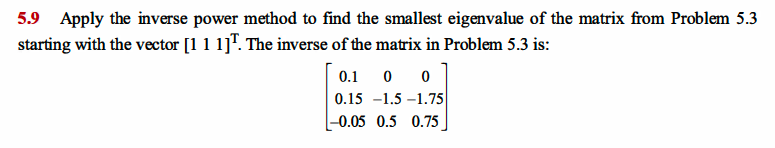
Therefore, the normalized unit vector of is

This is repeated and on our 6th iteration we get:

Since the difference between the multiplicative factors (was 3.01 for ) is relatively small we can terminate our iterations here at .

We can also say that the largest Eigenvalue for the given matrix a is:

**3.0165**



For , the Eigen vector that is:

Calculate the next Eigen vector for

We must then extract the highest element from the vector and normalise the vector:

Therefore, the normalized unit vector of is

We then repeat these steps, for the next Eigen vector for

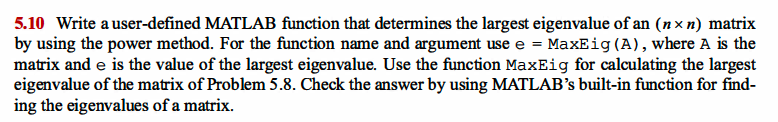
We must then extract the highest element from the vector and normalise the vector:

Therefore, the normalized unit vector of is

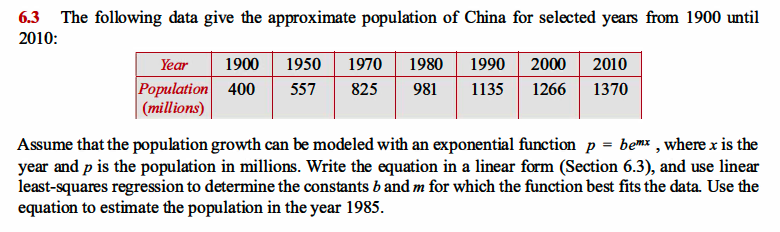
These steps are repeated until the difference between the multiplicative factors becomes significantly small. In our case this happens to be when ’s multiplicative factor is -1 and ’s multiplicative factor is -1.0002. We can terminate the iterations here and confidently say this is approximately the largest Eigenvalue and Eigenvector.

We can also say that the largest Eigenvalue for the given matrix a is:

**-1**

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Consider the function:

Apply the natural logarithm on both sides to bring down the exponent:

Now, define the terms as follows:

We can now write the equation in linear form as:

Re-write the data in the table taking the natural logarithm of each entry:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Year (x) | 1900 | 1950 | 1970 | 1980 | 1990 | 2000 | 2010 |
| Population ln(p) = Y | 5.9915 | 6.3226 | 6.7154 | 6.8886 | 7.0344 | 7.1436 | 7.2226 |

Now, apply the least square regression method. First, calculate the value of

Then, calculate the value of

Then, calculate the value of

Then, calculate the value of

Then, calculate the value of m

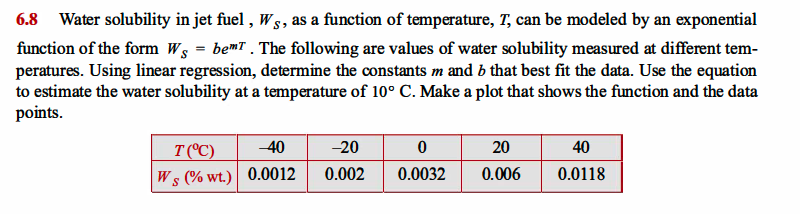
Then, calculate the value of B

Now, using our original equations solve the value of b:

Therefore, using our newfound values for b and m, the new relation is:

We can then, using this determine an estimation of the population in the year 1985 by letting x equal to 1985 in the above relation:

Therefore, we can estimate that the population in 1985 will be approximately 975.7718 million



Consider the function:

Apply the natural logarithm to both sides:

Let:

We can now express our function as a linear expression (let T =X):

Re-write the data in the table taking the natural logarithm of each entry:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Temp (X) | -40 | -20 | 0 | 20 | 40 |
| ln(Ws) (Y) | 6.7254 | 6.2146 | 5.7446 | 5.1160 | 4.4397 |

Now, apply the least square regression method. First, calculate the value of

Then, calculate the value of

Then, calculate the value of

Then, calculate the value of

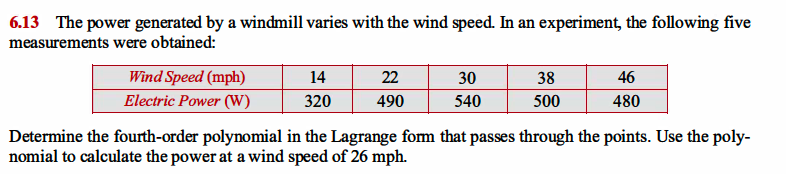
Then, calculate the value of m

Then, calculate the value of B

Now, using our original equations solve the value of b:

Therefore, using our newfound values for b and m, the new relation is:

We can then, using this determine an estimation of the water solubility in jet fuel at a temperature of 10° C:



The n-1th order Lagrange polynomial passing through n points can be defined as:

This can also be written as:

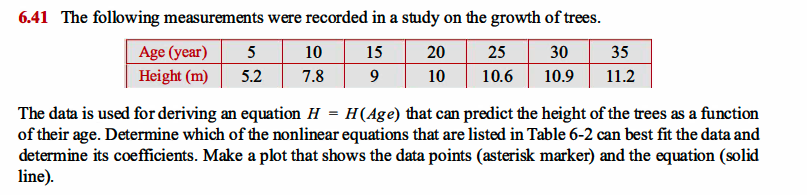
Therefore, the 4th order Lagrange polynomial passing through the five points and is:

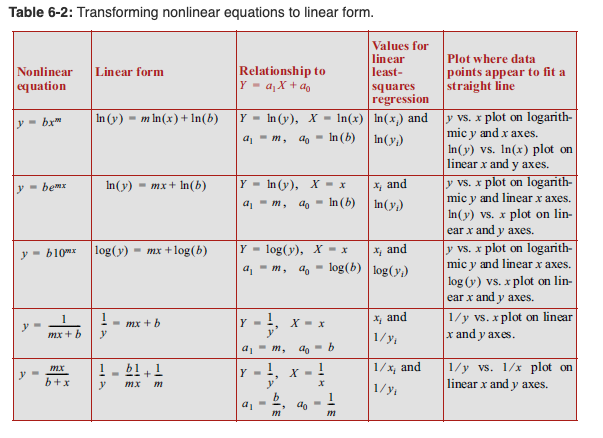
Using the data from the supplied table we then sub in the values for the following:

When we multiply this out, we are then left with the fourth order Lagrange polynomial as follows:

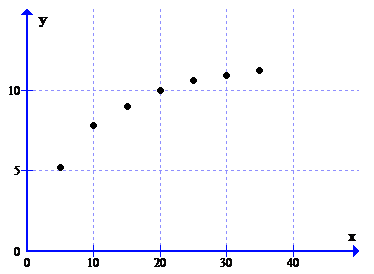
We can then use this polynomial to estimate the power at a wind speed of 26mph:

Thus, using the Lagrange fourth order polynomial we can estimate that the power at a wind speed of 26mph is approximately 528.18W.





In order to choose which function would be best suited to model the above data points, first plot x vs y:



The scatter plot shows that the graph is similar to the nonlinear equation from the table:

The linear form of this equation is:

We can then consider this:

And from that, the equation can be considered linear as:

Now, apply the least square regression method. First, calculate the value of

Since X = 1/x, we then have:

Then, calculate the value of

Since Y = 1/y, we then have:

Then, calculate the value of

Then, calculate the value of

Then, calculate the value of M

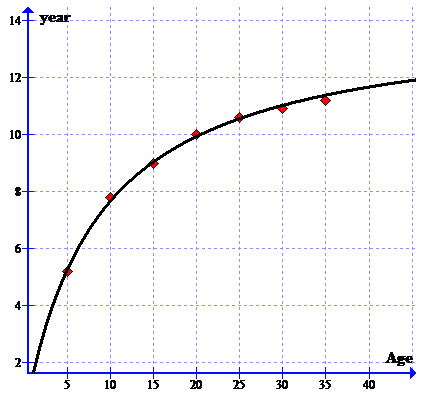
Then, calculate the value of C

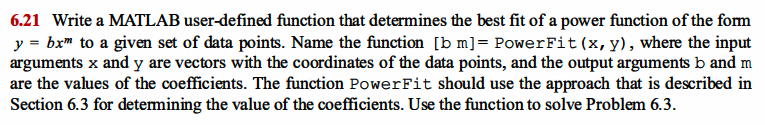
Now, using our original equations solve the value of m:

We can also solve the value of b:

Thus, we can now write our equation as:

By plotting the data points, and this given equation we can show that we have defined an accurate function to represent H=H(age):

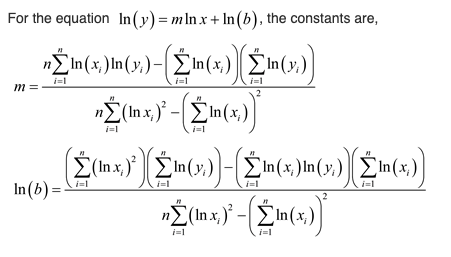




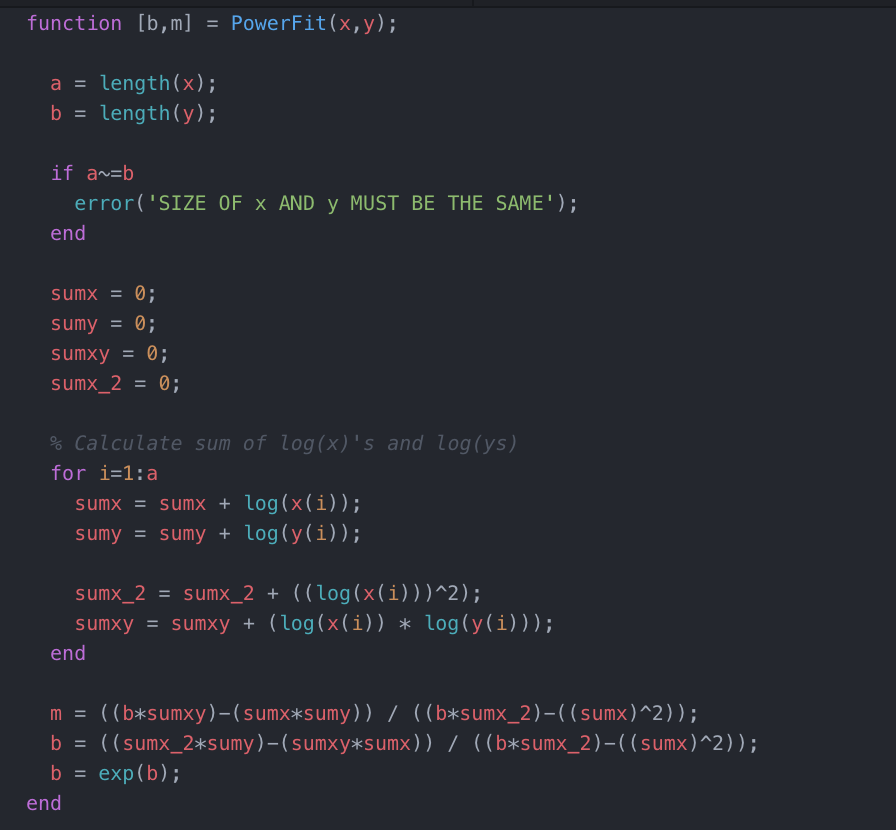
The exponential function is as follows:

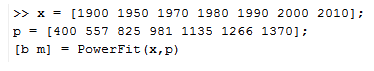
This equation can be written in linear form as follows:

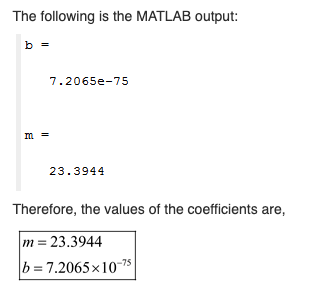
Now what we need to do is to determine the constants m and b. For a linear equation y = mx + b the coeffecients are:

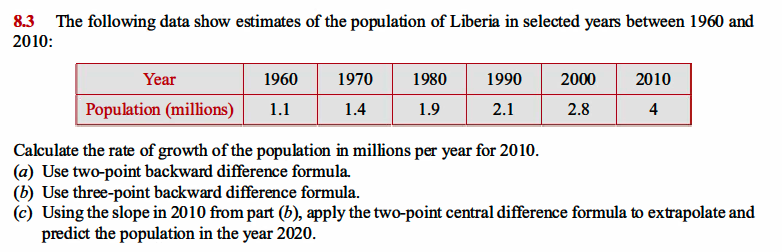


The MATLAB function PowerFit can be defined as follows:









The formula for the two point backward difference formula for the first derivative is as follows:

We can neglect O(h) here. Then, substitute 2010 for and 2000 for and 10 for h (distance between points):

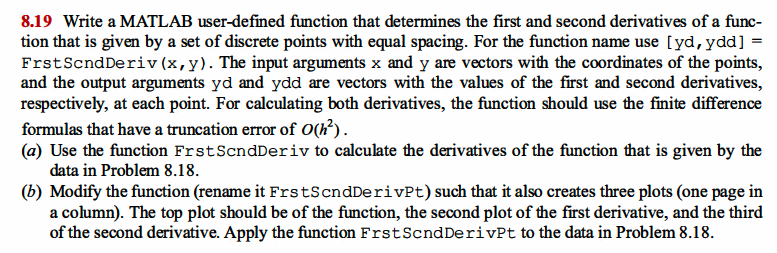
The formula for the three point backward difference formula for the first derivative is as follows:

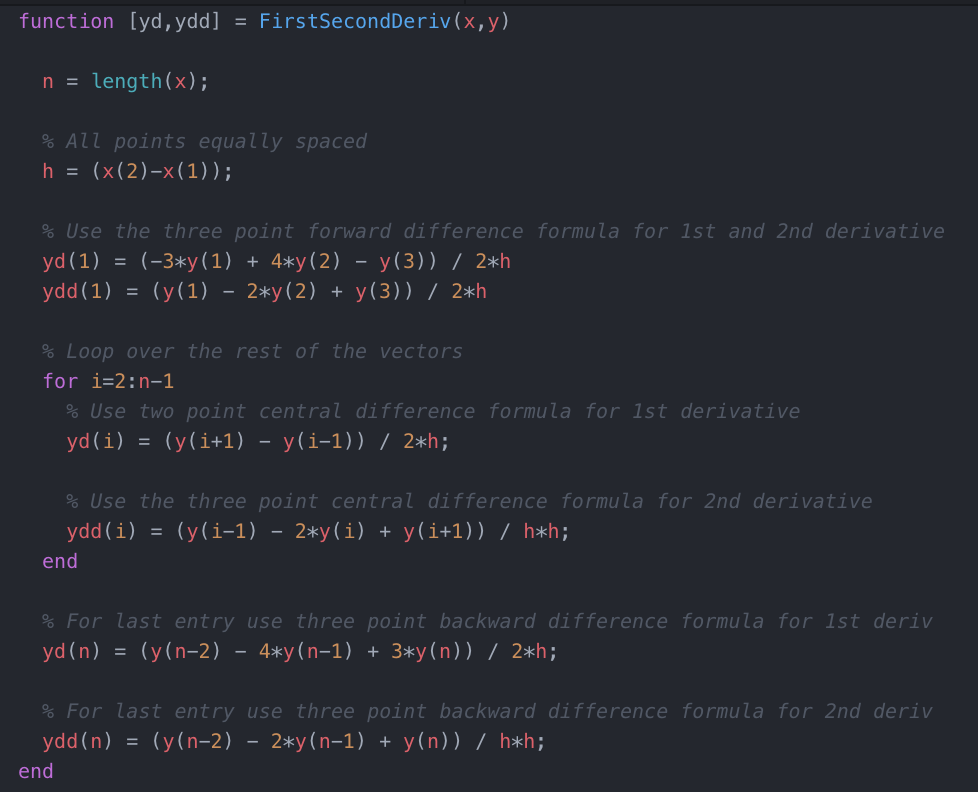
We can neglect O(h) here. The, substitute 2010 for , 2000 for , 1990 for and 10 for h (distance between points):

The formula for the two point central difference formula for the first derivative is as follows:

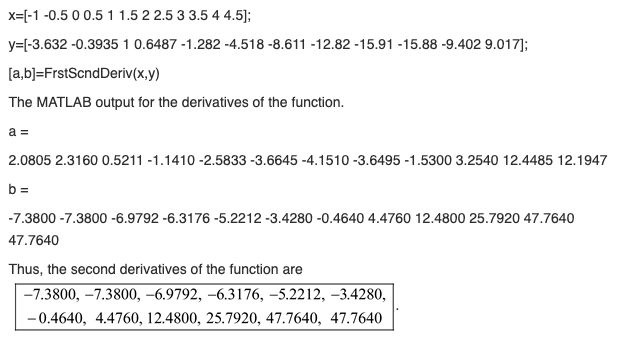
We can neglect O(h) here. The, substitute 2010 for , for , 1990, 10 for h (distance between points) and we will solve for :

Therefore, our prediction for the population of Liberia in 2020 is approximately 5.7 million.

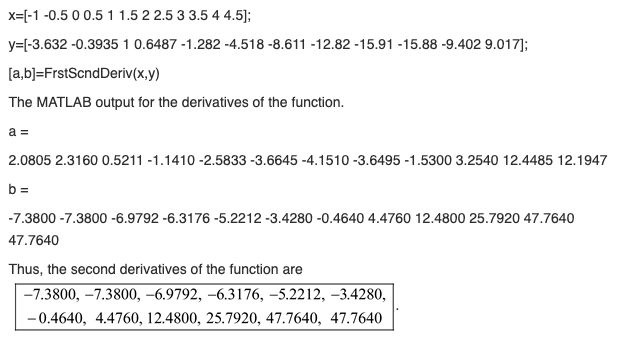




To then test this code to get the first and second derivatives do the following:



To then graph the results we would do the following:



This then produces the plots:

